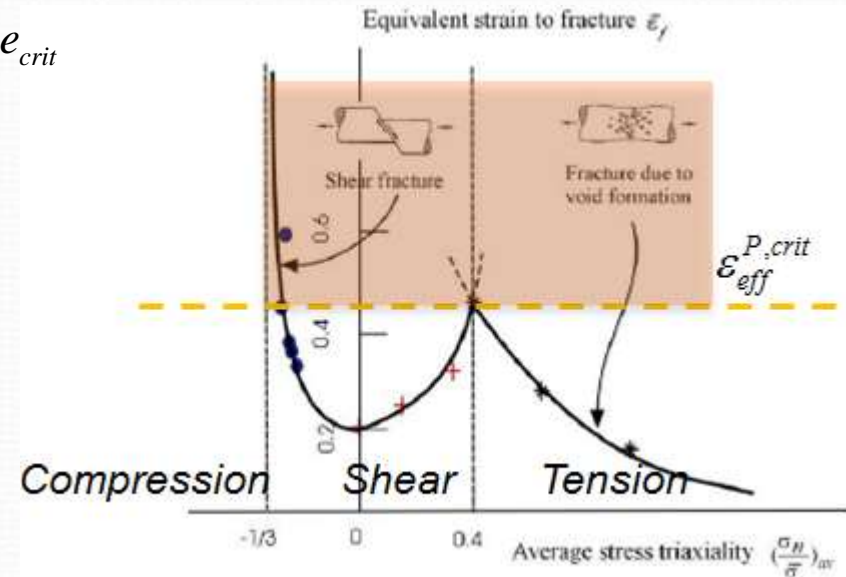# SPG Formulations:
Bond Failure Mechanism

$$\Phi_K(\mathbf{x}_J) = \begin{cases} 0 & \text{if } \delta > \delta_{crit} \text{ and } e > e_{crit} \\ \sum_{I=1}^{NP} \Psi_I(\mathbf{x}_J)\Psi_K(\mathbf{x}_I) & \text{Otherwise} \end{cases}$$
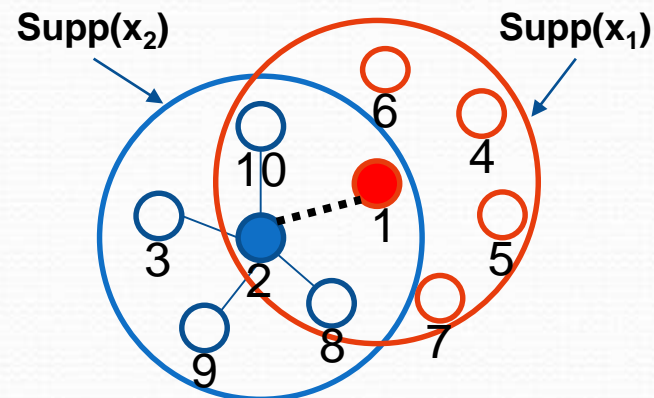
$$\varepsilon_{KJ}^p = \frac{\varepsilon_{eff}^P(\mathbf{X}_K) + \varepsilon_{eff}^P(\mathbf{X}_J)}{2} \qquad e = \frac{\|\mathbf{x}_J - \mathbf{x}_I\|}{\|\mathbf{X}_J - \mathbf{X}_I\|}$$

δ could be:
  Material damage
  <u>Effective plastic strain</u>
  First principal strain
  Maximum shear strain
  First principal stress
  Maximum shear stress
  Maximum tensile pressure

$\emptyset_2(x_1)=0$
$\emptyset_2(x_3)\neq0$
$\emptyset_1(x_2)=0$
$\emptyset_1(x_4) \neq 0$

SPG Failure vs FEM Failure

	FEM Failure	SPG Failure
Criteria	Effective plastic strain Add erosion Mesh dependent	Effective plastic strain Add erosion Insignificant discretization dependence
Upon failure	Zero stress Possible element deletion	Regular stress – strain evolution Bond failure without element erosion
Momentum	Not conserved	Conserved
Mass	Might not conserved	Conserved
Force	Under estimated	Physical

$$\mathbf{M}\Delta\ddot{\mathbf{u}} = \mathbf{f}^{ext} - \mathbf{f}^{int}$$

$$\mathbf{f}_I^{int} = \int_{\Omega_I} \mathbf{B}_I^T \mathbf{\Sigma} \, d\Omega$$

$$\mathbf{\Sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{22} & \sigma_{33} & \sigma_{12} & \sigma_{23} & \sigma_{31} \end{bmatrix}^T$$